Modeling the Effects of Taxes on Asset Sales and Borrowing

In this Appendix, we describe a simple model of how taxes affect the decision to finance consumption spending through asset sales (i.e. dissaving) or borrowing.

Assume that a person wishes to obtain funds C at time $t = 0$. They own \$1 of assets and will consume out of C for n years until death at time $t + n$, at which point the value of their estate V is given to an heir. (C is not invested.)

Consumption C is financed by borrowing B or asset sales S, less any taxes T associated with borrowing or asset sales:

$$C = B + S - T \tag{1}$$

Taxes are the sum of deemed realization taxes on borrowing T_B, withholding taxes on borrowing T_W, and capital gains taxes on asset sales T_S.

$$T = T_B + T_W + T_S \tag{2}$$

Taxes on borrowing B are structured as a deemed realization of gains at a rate of τ_B, assuming average basis.

$$T_B = B(1 - b)\tau_B \tag{3}$$

Withholding taxes on borrowing B are levied on the gross proceeds at a rate of τ_W. (Equivalently, this is a capital gains tax assuming basis of 0.)

$$T_W = B\tau_W \tag{4}$$

Taxes on asset sales S are levied with a normal realization capital gains tax at a rate of τ_S, assuming average basis.

$$T_S = S(1 - b)\tau_S \tag{5}$$

Consumption C, then, can be expressed as:

$$\begin{aligned} C &= B + S - (B(1 - b)\tau_B + B\tau_W + S(1 - b)\tau_S) \\ &= B + S - B(1 - b)\tau_B - B\tau_W - S(1 - b)\tau_S \\ &= B(1 - (1 - b)\tau_B - \tau_W) + S(1 - (1 - b)\tau_S) \end{aligned} \tag{6}$$

Rearranging the consumption equation for borrowing and sales respectively, we get the "tax gross-up" terms for B and S. In other words, nonzero taxes on sales/borrowing increase the amount of sales/borrowing required to finance a given value of C.

$$B = \frac{C - S(1 - (1 - b)\tau_S)}{1 - (1 - b)\tau_B - \tau_W} \tag{7}$$

$$S = \frac{C - B(1 - (1 - b)\tau_B - \tau_W)}{1 - (1 - b)\tau_S} \tag{8}$$

The person lives an additional n years after $t = 0$. During this time, their assets grow at a real rate of return r with inflation rate π. This means that, at death, the agent has assets A equal to:

$$A = (1 - S)e^{(r+\pi)n} \tag{9}$$

The person's debt is financed at real rate i, with debt rolling over until repayment at death. There is also an additional annual excise tax rate τ_e on the stock of debt. So, at death, liabilities L are equal to:

$$L = Be^{(i+\pi+\tau_e)n} \tag{10}$$

Finally, at death, the agent faces capital gains tax T_D on any unrealized gains that were borrowed against during lifetime, but did not face tax. The tax rate is τ_D. Crucially, to avoid double-counting, taxes already paid on borrowing (T_B and T_W) are credited against T_D:

$$T_D = (e^{(r+\pi)n} - b)\tau_D + B(1-b)\tau_D - T_B - T_W \tag{11}$$

So the final value V of the estate is:

$$V = A - L - T_D \tag{12}$$

And its present value is:

$$PV(V) = Ve^{(-i+\pi)n} \tag{13}$$

We can calculate an effective tax rate (ETR) metric by comparing the post-tax present value of the estate and present value of the estate under a no-tax counterfactual, then dividing by consumption. In the no-tax counterfactual, non-consumed assets grow at rate r and are discounted at rate i. Debt is rolled over at a rate of i but is discounted at the same rate. Denoting the agent's estate in this world as V', we have:

$$PV(V') = (1-S)e^{(r-i)n} - B \tag{14}$$

The ETR, then, is given by:

$$ETR = \frac{((1-S)e^{(r-i)n} - B) - Ve^{(-i+\pi)n}}{C} \tag{15}$$

And a notion of the tax advantage for borrowing ($C = S - T$) over sales ($C = B - T$) can be defined as:

$$\Delta = ETR_{sale} - ETR_{borrow} \tag{16}$$