# The Budget Lab's "Economic Value" Welfare Measure: A Second-Order Approximation to Equivalent Variation for Policy Changes

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## 1 Environment

Agent $i$ chooses a vector of $K$ consumption goods $x_i \equiv (x_{i1}, \ldots, x_{iK})$ and labor supply $l_i$ to maximize utility $u_i(x_i, l_i)$ subject to the budget constraint

$$I_i + y_i + w_i l_i - T(y_i + w_i l_i) = \sum_k \left[p_k + \tau_k(y_i + w_i l_i)\right] x_{ik}, \tag{1}$$

where:

- $I_i$ is exogenous nontaxable income;[1]
- $y_i$ is exogenous taxable income;
- $w_i$ is the (before-tax) wage;
- $p_k$ is the (before-tax) price of good $k$;
- $T(\cdot)$ is a (possibly negative) nonlinear income tax; and
- $\tau_k(\cdot)$ is a tax/subsidy on good $k$ that may depend on income (including, e.g., income-dependent subsidies with $\tau_k < 0$).

Define gross taxable income as $z_i \equiv y_i + w_i l_i$. The agent's indirect utility given the primitives is $v_i(I_i, y_i, w_i, p, T, \tau)$.

## 2 Policies

A policy may change any of $(I_i, y_i, p, w_i, T(\cdot), \tau(\cdot))$, either directly (as in the case of the tax schedules) or indirectly via equilibrium effects (in the case of prices and wages). Denote baseline (pre-policy) values with superscript 0 and scenario (post-policy) values with superscript 1. Let $(x_i^0, l_i^0)$ and $(x_i^1, l_i^1)$ be the agent's choices at the baseline and scenario equilibria, respectively.

The **equivalent variation** for the policy change, $E$, is defined implicitly by:

$$v_i(I_i^0 + EV_i,\ y_i^0,\ w_i^0,\ p^0,\ T^0,\ \tau^0) = v_i(I_i^1,\ y_i^1,\ w_i^1,\ p^1,\ T^1,\ \tau^1). \tag{2}$$

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[1] The use of $I_i$ is largely for notational simplicity when denoting income effects; it is without loss of generality to assume $I_i = 0$ in any equilibrium.

That is, $EV_i$ is the change in nontaxable income at baseline prices and tax schedules that would make the agent indifferent between the baseline and the scenario. Since $I_i$ is nontaxable, $EV_i$ is denominated in after-tax dollars.

We assume that all equilibrium quantities $(x_i^0, l_i^0, x_i^1, l_i^1)$, prices $(p^0, w_i^0, p^1, w_i^1)$, exogenous incomes $(I_i^0, y_i^0, I_i^1, y_i^1)$, and tax schedules $(T^0, \tau^0, T^1, \tau^1)$ are observed.

## 3 Second-Order Approximation to Equivalent Variation

We show here that the equivalent variation of individual $i$ can be approximated to second order by:

$$EV_i \approx \frac{1}{2}\mathcal{M}_i(x_i^0, l_i^0) + \frac{1}{2}\mathcal{M}_i(x_i^1, l_i^1) + \mathcal{R}_i, \tag{3}$$

where the mechanical component is given by

$$\begin{aligned}\mathcal{M}_i(x, l) = (y_i^1 - y_i^0) + w^1 l - w^0 l - x \cdot (q^1 - q^0) \\ - \Big[T^1(y_i^1 + w^1 l) - T^0(y_i^0 + w^0 l)\Big],\end{aligned} \tag{4}$$

and the income-effects correction term is

$$\mathcal{R}_i = -\frac{1}{2}\left(I_i^1 - I_i^0\right)\left[\eta_{l,I}^0 s_{li}^0 + \eta_{l,I}^1 s_{li}^1\right]. \tag{5}$$

*Proof.* We begin from the money-metric representation of equivalent variation:

$$EV_i = \int_0^1 \frac{1}{\lambda_i^\theta}\frac{dv_i^\theta}{d\theta} d\theta, \tag{6}$$

where $\lambda_i^\theta = \partial v_i^\theta / \partial I$ denotes the marginal utility of income.

**Step 1: Envelope representation** Using the envelope theorem, the derivative of indirect utility along a smooth policy path satisfies:

$$\frac{1}{\lambda_i^\theta}\frac{dv_i^\theta}{d\theta} = -x_i^\theta \cdot \frac{dq^\theta}{d\theta} + l_i^\theta \frac{dw^\theta}{d\theta} + \frac{dy_i^\theta}{d\theta} - \frac{dT^\theta(z_i^\theta)}{d\theta}, \tag{7}$$

where $q^\theta = p^\theta + \tau^\theta(\cdot)$ and $z_i^\theta = y_i^\theta + w^\theta l_i^\theta$.

**Step 2: Rearrangement** Rewriting terms yields:

$$\frac{1}{\lambda_i^\theta}\frac{dv_i^\theta}{d\theta} = \frac{dz_i^\theta}{d\theta} - x_i^\theta \cdot \frac{dq^\theta}{d\theta} - \frac{dT^\theta(z_i^\theta)}{d\theta} - w^\theta \frac{dl_i^\theta}{d\theta}. \tag{8}$$

Integrating over $\theta \in [0, 1]$ gives:

$$EV_i = \int_0^1 \left[\frac{dz_i^\theta}{d\theta} - x_i^\theta \cdot \frac{dq^\theta}{d\theta} - \frac{dT^\theta(z_i^\theta)}{d\theta} - w^\theta \frac{dl_i^\theta}{d\theta}\right] d\theta. \tag{9}$$

**Step 3: Mechanical components** Applying the fundamental theorem of calculus and trapezoidal approximation yields:

$$\int_0^1 x_i^\theta \cdot \frac{dq^\theta}{d\theta} d\theta \approx \frac{1}{2}\left[x_i^0 + x_i^1\right](q^1 - q^0), \tag{10}$$

$$\int_0^1 \frac{dT^\theta(z_i^\theta)}{d\theta} d\theta \approx \frac{1}{2}\Big[T^1(z_i^0) - T^0(z_i^0) + T^1(z_i^1) - T^0(z_i^1)\Big]. \tag{11}$$

Labor income terms are decomposed as:

$$-\int_0^1 w^\theta \frac{dl_i^\theta}{d\theta} d\theta = -(w^1 l_i^1 - w^0 l_i^0) + \int_0^1 l_i^\theta \frac{dw^\theta}{d\theta} d\theta. \tag{12}$$

Applying trapezoidal approximation:

$$\int_0^1 l_i^\theta \frac{dw^\theta}{d\theta} d\theta \approx \frac{1}{2}(l_i^0 + l_i^1)(w^1 - w^0). \tag{13}$$

Collecting terms yields the symmetric two-point representation in $\mathcal{M}_i(\cdot)$.

**Step 4: Income effects** Decompose labor supply response:

$$\frac{dl_i^\theta}{d\theta} = \underbrace{\text{substitution effects}}_{\text{absorbed in mechanical terms}} + \frac{\partial l_i}{\partial I}\frac{dI_i^\theta}{d\theta}. \tag{14}$$

The remaining term becomes:

$$\mathcal{R}_i = -\int_0^1 w^\theta \frac{\partial l_i}{\partial I}\frac{dI_i^\theta}{d\theta} d\theta. \tag{15}$$

Define the income elasticity:

$$\eta_{l,I}^\theta = \frac{\partial l_i}{\partial I}\frac{I_i^\theta}{l_i^\theta}, \quad s_{li}^\theta = \frac{w^\theta l_i^\theta}{I_i^\theta}. \tag{16}$$

Then:

$$\mathcal{R}_i = -\int_0^1 \eta_{l,I}^\theta s_{li}^\theta \frac{dI_i^\theta}{d\theta} d\theta. \tag{17}$$

Applying the trapezoidal rule:

$$\mathcal{R}_i \approx -\frac{1}{2}(I_i^1 - I_i^0)\left[\eta_{l,I}^0 s_{li}^0 + \eta_{l,I}^1 s_{li}^1\right]. \tag{18}$$

□

# 4 Defining the “Economic Value” of a Policy Change

Neglecting the income-effect residual term of the approximation to the equivalent variation, the remaining terms of equation (3) can be rewritten as:

$$
\begin{aligned}
EV_i \equiv & \frac{1}{2}\Bigg[\Delta I_i + \Delta y_i + l_i^0 \Delta w_i - \sum_k x_{ik}^0 \Delta p_k \\
& - \left[T^1\left(y_i^1 + w_i^1 l_i^0\right) - T^0\left(y_i^0 + w_i^0 l_i^0\right)\right] \\
& - \sum_k x_{ik}^0 \cdot \left[\tau_k^1\left(y_i^1 + w_i^1 l_i^0\right) - \tau_k^0\left(y_i^0 + w_i^0 l_i^0\right)\right]\Bigg] \\
+ & \frac{1}{2}\Bigg[\Delta I_i + \Delta y_i + l_i^1 \Delta w_i - \sum_k x_{ik}^1 \Delta p_k \\
& - \left[T^1\left(y_i^1 + w_i^1 l_i^1\right) - T^0\left(y_i^0 + w_i^0 l_i^1\right)\right] \\
& - \sum_k x_{ik}^1 \cdot \left[\tau_k^1\left(y_i^1 + w_i^1 l_i^1\right) - \tau_k^0\left(y_i^0 + w_i^0 l_i^1\right)\right]\Bigg],
\end{aligned}
$$

This is the **economic value** of the policy change.